Section 2.1

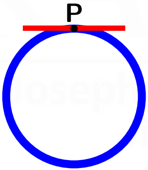
Limit Idea: Instantaneous Velocity and Tangent Lines

(1) Tangent Lines
(2) Secant Lines
(3) The Velocity Problem
(4) The Tangent Problem

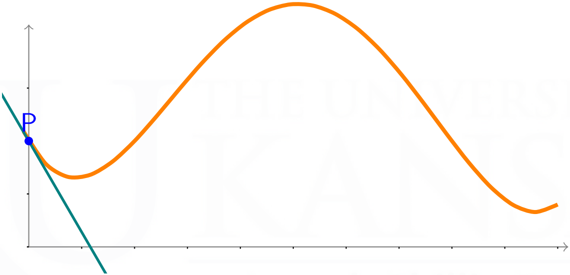
Tangent Lines

Tangent Lines

The **Tangent Line** to a curve at the point **P** is the line that "just touches" the curve at **P**.



Tangent Lines



Tangent Lines

The **Tangent Line** to a curve at the point **P** is the line that "just touches" the curve at **P**.

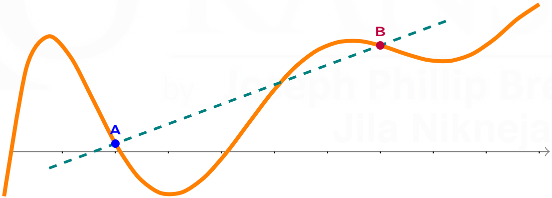
Secant Lines

Secant Lines

The **Secant Line** to a curve **F** at the points **A** and **B** is the line that passes through **A** and **B**.

The slope of the secant line is

$$m = \frac{\Delta y}{\Delta x} = \frac{F(b) - F(a)}{b - a}$$

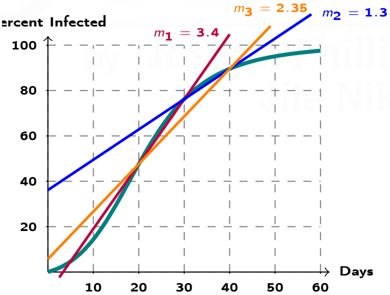


▸ Link

A computer virus has been released which spreads through a common voicemail application preloaded on many smartphones. The following table describes the spread of this virus:

Days	0	10	20	30	40	50	60
Percent Infected	0	16	44	78	91	96	98

Find the average rate of change of infection over the intervals $[20, 30]$, $[30, 40]$, and $[20, 40]$. Explain this growth with secant lines on the graph.



The Velocity Problem

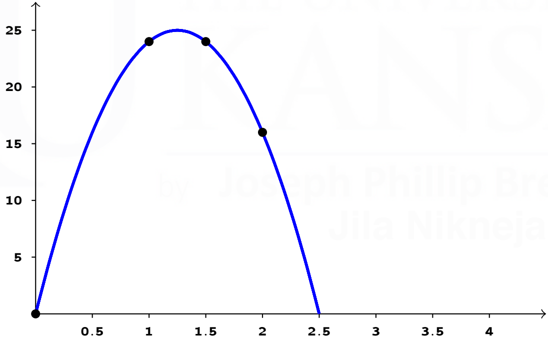
Velocity

Velocity is a type of Rate of Change.

Units: $\frac{miles}{hour}$ $\frac{kilometers}{hour}$ $\frac{feet}{second}$ $\frac{meters}{second}$

Average Velocity

A ball is thrown upwards with a velocity of 40 feet per second. The height in feet t seconds later is given by $y = 40t - 16t^2$. Find the average velocity of the ball between times $[0, 2]$, $[1, 2]$, and $[1.5, 2]$.



The Tangent Problem

Given a point **P** on a function **F**, how do we find the **tangent line** of **F** at **P**? ▸ Link

Tangent Line

The **tangent line** to the function $y = f(x)$ at a point P is a secant line through the point P and a point infinitely close to P on the curve.

Tangent Slope

$$\lim_{b \to a} \frac{f(b) - f(a)}{b - a} \qquad \lim_{h \to 0} \frac{f(a+h) - f(a)}{(a+h) - a}$$

In a game of Quidditch at Hogwarts, a ball is thrown upwards and the heights at certain times have been recorded:

Time (seconds):	2	2.5	2.9	2.95	2.995	3
Height (feet):	20	31.25	42.05	43.51	44.8501	45

Suppose the relationship between time and height is represented by the function $H = F(T)$. Can we graph the function using the table?

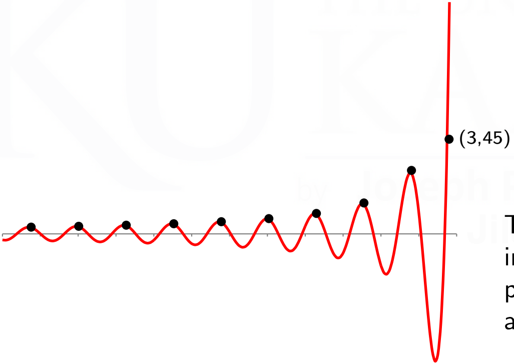
If we were interested in the tangent line at time $T = 3$ seconds we already have a point, (3,45), on the line, but we don't have the **slope**!

Can we approximate it by looking at the trend of the average velocities?

Interval:	[2,3]	[2.5,3]	[2.9,3]	[2.95,3]	[2.995,3]
Avg Velocity:	25	27.5	29.5	29.75	29.975

WHAT IF?????

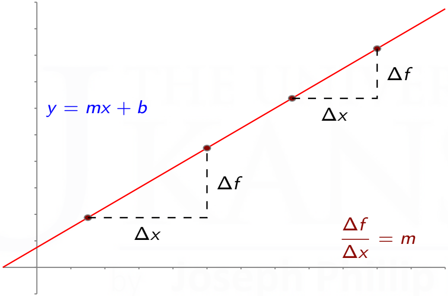
Interval:	[2,3]	[2.5,3]	[2.9,3]	[2.95,3]	[2.995,3]
Avg Velocity:	25	27.5	29.5	29.75	29.975



The Quidditch ball was bouncing. Table did not contain enough points on the graph. How do we avoid similar scenarios?

Linear Functions $f(x) = mx + b$

Linear functions are characterized by their uniform average rates of change.



The $\underbrace{\text{average rate of change for a linear function}}_{\text{slope of the line}}$ between **any** distinct pair of points is the slope m.

The $\underbrace{\text{instantaneous rate of change}}_{\text{slope of the line}}$ at **any** point is the slope m.